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TECHNICAL REPORT ARBRL-TR-02137

MECHANICS OF PENETRATION: ANALYSIS
AND EXPERIMENT

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February 1979

# US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
## BALLISTIC RESEARCH LABORATORY
### ABERDEEN PROVING GROUND, MARYLAND

| REPORT DOCUMENTATION PAGE | | READ INSTRUCTIONS BEFORE COMPLETING FORM |
|---|---|---|
| 1. REPORT NUMBER <br> TECHNICAL REPORT ARBRL-TR-02137 | 2. GOVT ACCESSION NO. | 3. RECIPIENT'S CATALOG NUMBER |
| 4. TITLE (and Subtitle) <br> MECHANICS OF PENETRATION: ANALYSIS AND EXPERIMENT | | 5. TYPE OF REPORT & PERIOD COVERED <br> Final |
| | | 6. PERFORMING ORG. REPORT NUMBER |
| 7. AUTHOR(s) <br> G. H. Jonas <br> J. A. Zukas | | 8. CONTRACT OR GRANT NUMBER(s) |
| 9. PERFORMING ORGANIZATION NAME AND ADDRESS <br> US Army Ballistic Research Laboratory <br> ATTN: DRDAR-BLT <br> Aberdeen Proving Ground, MD 21005 | | 10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS <br> RDT&E 1L162618AH80 |
| 11. CONTROLLING OFFICE NAME AND ADDRESS <br> US Army Armament Research and Development Command <br> US Army Ballistic Research Laboratory <br> ATTN: DRDAR-BL <br> Aberdeen Proving Ground, MD 21005 | | 12. REPORT DATE <br> FEBRUARY 1979 |
| | | 13. NUMBER OF PAGES <br> 64 |
| 14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) | | 15. SECURITY CLASS. (of this report) <br><br> UNCLASSIFIED |
| | | 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE |

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

kinetic energy
projectiles
analytical methods
numerical simulation

20. ABSTRACT (Continue on reverse side if necessary and identify by block number) (lrs)

This paper reviews available analytical methods for the study of kinetic energy (inert) projectile-armor interactions at ordnance velocities ($0.5$-$2$ $kms^{-1}$). Particular emphasis is placed on three-dimensional numerical simulation of perforation and results of two-dimensional plane strain and three-dimensional finite element computations are compared with experimental results for impact situations leading to perforation and to ricochet. Problem areas and requirements for improved materials characterization are discussed.

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# I. INTRODUCTION

The subject of penetration and perforation of solids has long been of interest in the military field and has recently commanded attention in a number of industrial applications, viz., the integrity of nuclear reactor pressure vessels and survivability of aircraft turbine blades struck by birds. Galileo was among the first to observe the difference between the static and dynamic behavior of metals. Serious research in analytical methods was undertaken about the time of World War II and continues to the present day; the current emphasis being on two- and three-dimensional numerical solutions to high velocity impact problems.

Penetration may be defined as the entrance of a missile into a target without completing its passage through the body. This involves either the embedding or rebound of the striker and the formation of a crater. Perforation, on the other hand, implies the complete piercing of a target by the projectile. Such processes tend to occur in a time frame of several to several hundred microseconds. The target can fail in a variety of ways, among them petalling, plugging, ductile failure and spall. Considerable deformation in the penetrator can also be expected. Some typical examples of penetrator-target interactions can be seen in Figures 1 through 4.

Since deformation is primarily determined by the velocity of impacting bodies, a short classification of impact processes as a function of striking velocity (and strain rate) seems appropriate (Table I). The range limitations should only be considered as reference points. In fact, the transitions are extraordinarily flexible since deformation processes depend on a long series of parameters in addition to impact velocity.

Penetration and perforation then are formidable physical problems. A rigorous analysis would require that one account for the geometry of the interacting bodies, elastic-plastic and shock wave propagation, hydrodynamic flow, finite strains and deflections, strain rate effects, work hardening, heating or frictional effects and the initiation and propagation of fracture. It is not surprising, therefore, that the bulk of the research in this area has been experimental in nature.

Analytical approaches have tended to fall into three categories:

- <u>empirical or quasi-analytical</u>: algebraic equations are formulated based on correlation with a large number of experimental data points and these are used to make predictions to guide further experiments. Such efforts are usually closely related to tests performed to discriminate between the performance characteristics of various materials or structures for a particular design objective. In general, these efforts do not significantly advance our understanding of material behavior and processes and will not be considered in this paper. A variety of such models for penetration and ricochet have been reviewed by

Figure 1. Tungsten Alloy Rod Striking Thin Steel Plate.

PLUG VELOCITY = 365 FT/SEC
PLUG WEIGHT = 7.2 GRAINS

TIME AFTER IMPACT, μSEC
507
357
227
127
52
0

STRIKING VELOCITY =
940 FT/SEC
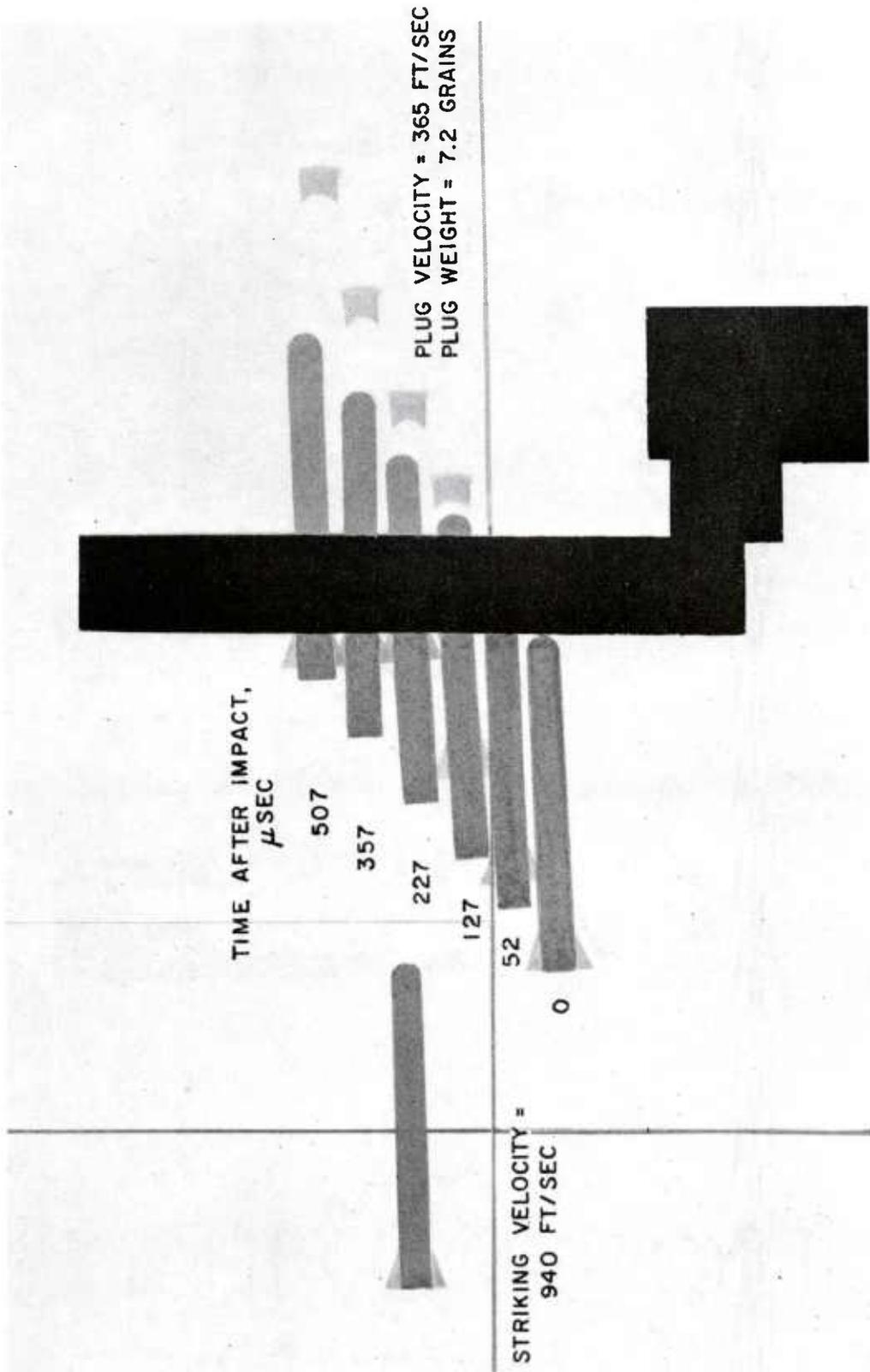
Figure 2. Example of Target Plugging Failure

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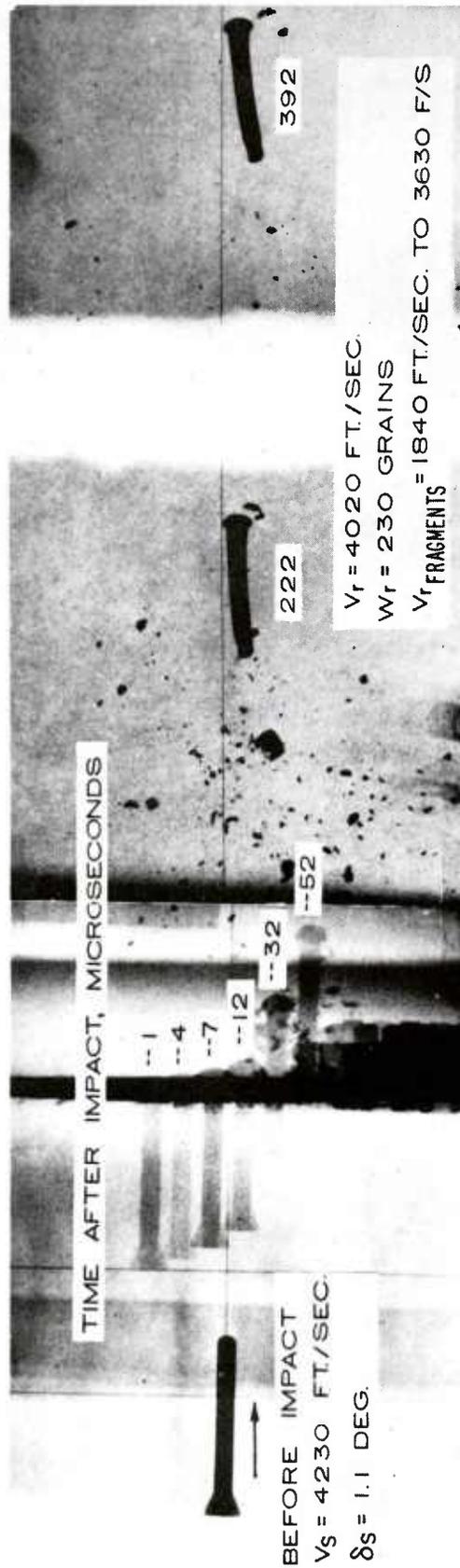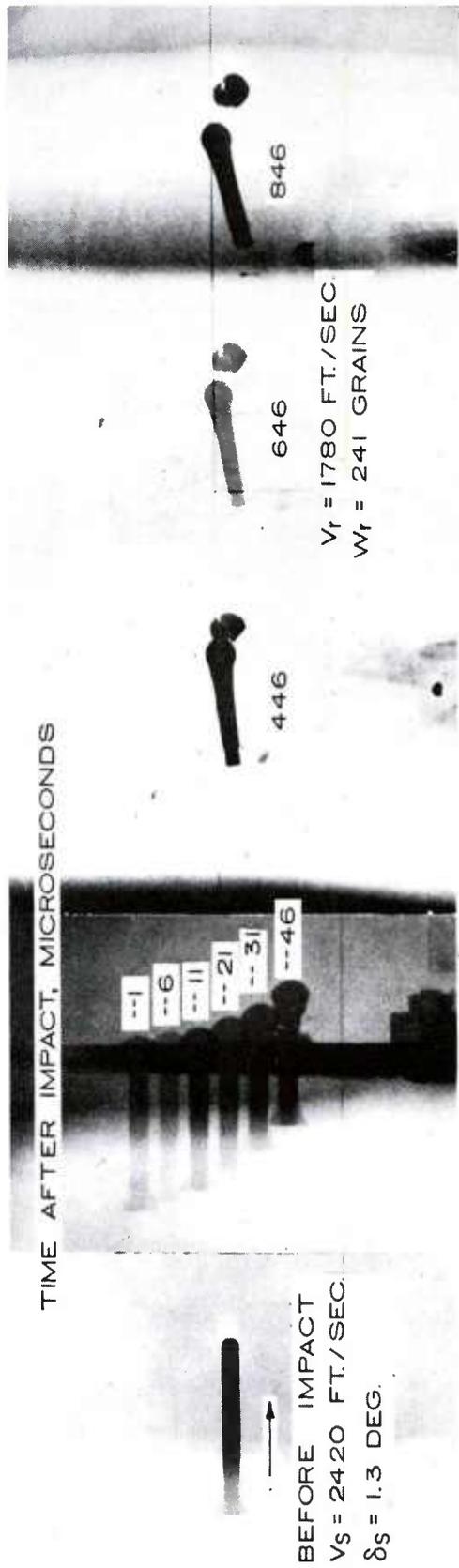
Figure 3. Penetrator-Target Deformation Variation with Impact Velocity

6A.3 GRAM VIMVAR-PROCESSED BEARCAT STEEL PENETRATOR,

D = 1.02 CM, L/D = 10 INTO 2.54 CM RHA AT 60°

$V_S$ = 1360 m/s
$V_R$ = 769 m/s
$M_R$ = 20.7 g
$\triangle$ = 96 g

$V_S$ = 1282 m/s
$V_R$ = 510 m/s
$M_R$ = 13.6 g
$\triangle$ = 97 g

$V_S$ = 1227 m/s
$V_R$ = 203 m/s
$M_R$ = 8.6 g
$\triangle$ = 68 g

$V_S$ = 1202 m/s
$V_R$ = 0 m/s
$M_R$ = 0 g
$\triangle$ = 14 g

$V_S$ = 1800 m/s
$V_R$ = 1285 m/s
$M_R$ = 13.6 g
$\triangle$ = 171 g

$V_S$ = 1647 m/s
$V_R$ = 1145 m/s
$M_R$ = 14.8 g
$\triangle$ = 156 g

$V_S$ = 1499 m/s
$V_R$ = 983 m/s
$M_R$ = 19.6 g
$\triangle$ = 126 g

$V_S$ = 1471 m/s
$V_R$ = 997 m/s
$M_R$ = 14.5 g
$\triangle$ = 98 g

Representative Original Penetrator

$M_R$: Residual Penetrator Mass
$\triangle$: Mass Loss of Target Plate
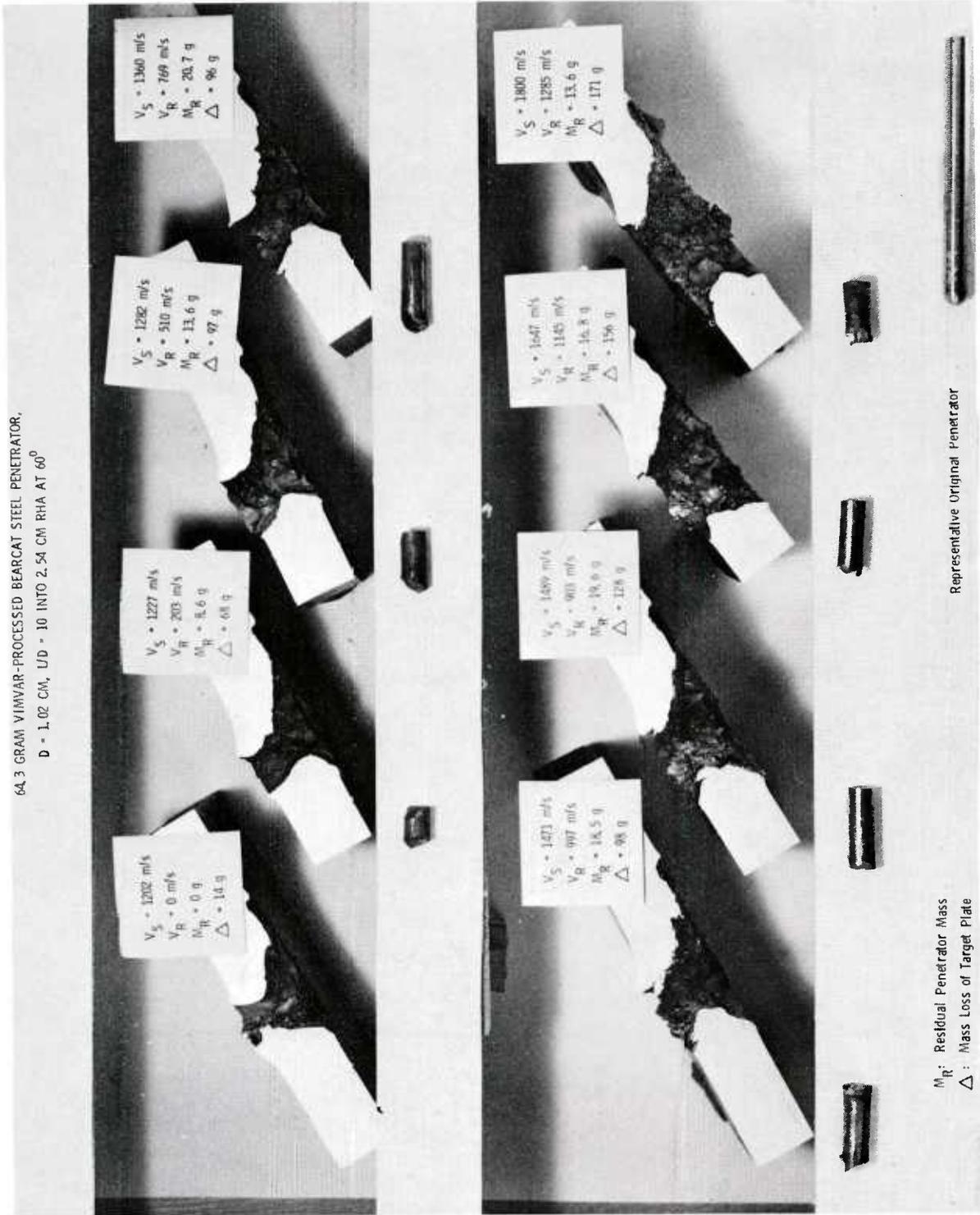
Figure 4. Sectioned Target Plates and Residual Penetrators

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Table I.  Impact Response of Materials

| $\dot{\varepsilon}$ | $V_s$ | Effect | Method of Loading |
|---|---|---|---|
| $10^8$ | >12 kms$^{-1}$ | Explosive impact- colliding solids vaporized | - |
| $10^7$ $10^6$ | 3-12 kms$^{-1}$ | Hydrodynamic- material compressi- bility not ignorable | Explosive acceleration |
| $10^5$ | 1-3 kms$^{-1}$ | Fluid behavior in materials; pressures approach or exceed material strength; density a dominant parameter | Powder guns, gas guns |
| $10^4$ $10^3$ | 500-1000 ms$^{-1}$ | Viscous-material strength still significant | Powder guns |
| $10^2$ $10^1$ | 50-500 ms$^{-1}$ | Primarily plastic | Mechanical devices, compressed air guns |
| $10^0$ $10^{-1}$ | <50 ms$^{-1}$ | Primarily elastic, some local plasticity | Mechanical devices, compressed air guns |
| 0 | | | |

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RECHT [1]. Similarity modeling for penetration mechanics is discussed in a chapter of the book by BAKER et al [2].

- approximate analytical methods: these concentrate on one aspect of the problem (such as plugging, petalling, spall, crater formation, etc.) by introducing simplifying assumptions into the governing equations of continuum physics in order to reduce these to one- or two-dimensional algebraic or differential equations. Their solution is then attempted, frequently in the course of which additional simplifications are introduced. With few exceptions, such analyses tend to treat either the striker or the target as rigid and rely on momentum or energy balance, or both. Only a few papers are concerned with predicting the deformation of both projectile and target. Furthermore, almost all such analyses either require some empirical input or rely on material parameters not readily available or measurable.

- numerical methods: for a complete solution of impact problems, one must rely on a numerical solution of the full equations of continuum physics. Finite difference and finite element methods are capable of attacking the entire set of field equations, have greater flexibility than various algebraic equations and can accurately model transient phenomena. They are still approximate in nature (one solves a set of discretized equations rather than the corresponding differential equations) but at present, errors associated with material properties are usually far greater than errors inherent in the numerical method.

This paper concerns itself with kinetic energy (inert) penetrator-armor interactions at ordnance velocities ($0.5 \leq V_s \leq 2$ km/s). No mention will be made of low velocity impacts and contact phenomena, which have been reviewed elsewhere [3-7], nor will problems in hypervelocity impact be considered. This subject has been treated extensively in various symposia on hypervelocity impact [8-15] and in reviews by HOPKINS and KOLSKY [16], HERRMANN and JONES [17], PITEK and HAMMIT [18], VINSON [19], KINSLOW [20], SWIFT [21] and W. JOHNSON [22]. In addition LEHTO [23] has compiled a bibliography of shock wave effects in solids. After a brief survey of current analytical and numerical efforts in kinetic energy penetration and perforation of armor at ordnance velocities, examples of current capabilities in numerically modeling oblique impact situations will be presented and compared to available experimental data. The strengths and weaknesses of current computer codes relevant to armor design will be discussed and areas requiring further research highlighted.

## II. ANALYTICAL MODELS

When a projectile strikes a finite thickness metal target, several possible types of damage may occur. The target may deform without fracture; penetration on axis may occur, followed by radial fracture of the target to allow projectile passage; shear failure can occur in the plate along an approximately cylindrical surface through the thickness of the plate such that a plug is pushed out ahead of the projectile; spallation (dynamic tensile fracture) can result at the rear surface of the target after impact on the front surface, caused by reflection of high intensity, short duration compressive stress pulses being reflected at the free surface of the body; or, penetration may occur by hydrodynamic flow of the target material. It has been generally noted in ballistic experiments that plugs tend to form in hard thick plates, dishing and petalling occur in thin ductile plates and ductile hole enlargement and spalling occur in softer thick plates.

The principal mode of failure will depend on the geometry and material properties of the projectile and target and on the velocity of the projectile. Frequently, a second or third mode accompanies the principal failure mode to a lesser extent.

Analytical models seek generally to predict the depth of penetration and crater dimensions for thick target penetration and the exit velocity or ballistic limit or both for thin target perforations. The various methods currently used for the determination of ballistic limits, together with their advantages and limitations, have been discussed by MISEY [24] and will not be elaborated upon here.

Most models consider either a single perforation mechanism (plugging, hole enlargement) or conservation law (energy, momentum). A few allow multiple perforation mechanisms, e.g., combinations of such factors as compression, plugging, tearing, target inertia, dynamic pressure, friction and drag. The models could also be subdivided as descriptive versus predictive, depending upon the degree of empiricism involved, but this point will not be pursued further here.

Several excellent surveys of ballistic impact modeling exist. NICHOLAS [25] reviewed some 245 articles dealing with ballistic impact from a mechanics viewpoint while GOLDSMITH [26] surveyed thin target perforation analysis for normal impact situations. Material covered in these surveys will be mentioned here only briefly when required for the sake of completeness.

Although penetration into semi-infinite targets at hypervelocity has received considerable attention, the literature for the same situation at ordnance velocities is quite limited. BROOKS [27] considers penetration and cratering processes for non-deforming projectiles impacting ductile targets. His model is based on an aerodynamic analogy, assuming a rigid-plastic "atmosphere". The model allows for an entry

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phase where the projectile tip is immersed in the target, a variation
of flow stress with increasing penetration depth, an elastic limit
below which permament hole enlargement does not occur and also allows
penetrators with doubly tapered conical nose tips. Predictions are
made for the shape of the crater and the depth of penetration and
comparisons made with experimental results for several projectile-target
material combinations. The model agreed with crater profiles for
steel and tungsten carbide penetrators striking soft aluminum, and
tungsten carbide penetrators striking mild steel and 4340 steel targets.
Among the principal findings of the study are the existence of a critical
value of a non-dimensional ballistic number (the ratio of the
instantaneous dynamic inertial pressure to local flow stress) below
which the crater diameter is equal to that of the projectile and the
observation that for a semi-apex angle greater than 55° there is
virtually no difference in the hole profiles produced at a given
velocity. The depth of penetration was found to vary strongly with
semi-apex angle, being some 50% greater for an angle of 10° than for 55°.

BROOKS [28] also advances a hypothesis to explain the behavior
of ductile projectiles striking thick targets. Analytical techniques
exist to treat impact of rigid projectiles and soft ones which flow
hydrodynamically. However, for deformable projectiles where material
strength is a significant factor, the deformation behavior is strongly
dependent on the impact velocity, the dynamic properties of the pro-
jectile and target materials and on the instantaneous shape of the
projectile. At a particular impact velocity, the projectile deformation
process will exhibit a dynamic instability that will change its behavior
from an essentially elastic character to one which is essentially
hydrodynamic. The velocity at which this occurs is termed the hydro-
dynamic transition velocity. On the basis of an extensive experimental
program, Brooks proposes the following deformation sequence:

(a) for all penetration velocities, target material is accelerated
radially away from the axis of penetration by the passage of the projectile.

(b) at low velocities, the kinetic energy imparted to this material
as lateral motion is recovered as elastic strain energy such that the
target material always remains in contact with the projectiles.

(c) as the impact velocity increases, the lateral acceleration
of the target material increases; the magnitude of the target kinetic
energy approaches the elastic strain energy and hence the degree of
hydrostatic support afforded the projectile by the target decreases.

(d) at the tip of the projectile, the slope of the surface relative
to the trajectory is greatest, hence the radial acceleration of the target
material near the tip of the projectile will be greater than at any other
point of the ogive. This effect will be even more pronounced if the
target material is of a type that hardens significantly when subjected
to plastic deformation.

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(e)   at the transition velocity, the kinetic energy imparted to the target material near the nose reaches a level at which it exceeds the elastic strain energy and the target can no longer provide hydrostatic constraint to prevent the projectile from deforming laterally.

(f)   once deformation of the point of the projectile is initiated, the process becomes unstable.  As the point becomes spherically blunted, the radial acceleration of the target material increases and exceeds that which would allow adequate hydrostatic support for the next element of projectile behind the deformed point.  The rate of deformation increases progressively and finally results in a total destruction of the ogive and the formation of a new dynamically stable profile.

Ballistic tests with ogival projectiles showed that the transition velocity varied inversely with tip radius, a result supporting the hypothesis.  It was also found experimentally that the transition velocity for a given shape can be forced to a higher level if tip deformation is inhibited by an appropriate selection of tip material.  Many of the salient points in Brooks' hypothesis are also supported by ZUKAS and JONAS [29] who studied numerically the effectiveness of ballistic caps of various materials for long rod penetrators.

TATE [30] suggests that the hydrodynamic transition velocity depends on the relative rates of rod erosion and plastic wave propagation.  He states that when the rod erosion rate exceeds the rate of propagation of gross plastic deformation, then all the gross deformation is constrained to occur very near the tip of the rod in a region of increased entropy and temperature resulting in a jet mode of penetration.  Expressions are given for the speed of propagation of gross plastic distortion and for the hydrodynamic transition velocity for right circular cylindrical rods striking thick targets at normal incidence in terms of the empirical dynamic strength of the rod and target and the dynamic work-hardening rate or dynamic large strain tangent modulus of the rod material.  A comparison with data for copper rods striking copper target indicates qualitative agreement.

TATE [31] proposes a modification to Bernoulli's equation by including two strength parameters (stresses for rod and target above which each behaves as a fluid) to predict deceleration of a long rod after striking a thick target.  The strength parameters are empirically determined quantities.  Comparison with experimental data indicates fair to poor agreement and a high degree of sensitivity of predicted results to the assumed values of the two strength parameters.  In additional developments along the above lines, TATE [32] provides models to account for deformation of a soft rod striking a rigid target and the penetration of a rigid projectile into a soft target. He shows that it is theoretically possible to have a decrease in depth of penetration with increasing impact velocity, but predictions are not substantiated by the experimental results cited.

BYRNSIDE, TORVIK and SWIFT [33] have had considerable success in studying penetration processes and their work merits further consideration for generalization and extension to the oblique impact case. Their approach is a modification of the Rigid Penetrator and Deep Penetration theories of GOODIER [34] to account for projectile strength in crater formation. It is assumed that a spherical projectile is not fragmented while being completely consumed. Experiments were performed with 7075-T6 aluminum projectiles striking 6061-H aluminum targets. Measured values of crater diameter and crater depth agreed quite well (0-8% deviation for mean crater diameter) with predicted values for velocities under 2 kms$^{-1}$. For increasing striking velocity and projectile strength, discrepancies between predictions and data increased (13% for hypervelocity data).

PERSSON [35] has developed a simple model for response of a relatively thick target normally impacted by a rigid sphere. Projectile motion is retarded by elastic-plastic, friction and inertia forces with provision for reduction of retarding forces due to edge and rear surface effects. Since the model comes with no less than eight adjustable parameters which must be determined by separate experiments, its utility is limited to connecting data points for those researchers lacking French curves.

Perforation of finite thickness plates has achieved greater attention. FUGELSO et al [36-38] survey at length the theoretical aspects of penetration and perforation and justify the use of linear elastic solutions to perforation problems for very thin plates at striking velocities under 1.2 kms$^{-1}$ and impact durations of less than 50 microseconds. Fracture is based on a critical octahedral shear stress. No comparison with experiments is made. FLORENCE and AHRENS [39] and FLORENCE [40] offer a linear elastic analysis of stresses in metal projectiles and ceramic targets. The experimental data presented is impressive.

Hole growth in ductile targets has been considered by BETHE [41], G. I. TAYLOR [42], FREIBERGER [43] and KUMARI [44]. W. T. THOMPSON [45] and BROWN [46] use a quasi-dynamic energy approach to study petalling of thin plates. KUCHER [47-48] optimizes penetrator shape using Thompson's theory. These approaches are conceptually interesting and permit considerable mathematical manipulations but are not very useful since few armor designs use infinitely thin sheets. GOLDSMITH [26] comments on analytical treatment of hole growth at some length.

In the energy balance analyses cited above, wave propagation effects, crack formation, friction, adiabatic heating and strain rate are not considered. ZAID and PAUL [49] assumed that penetration effects propagate at a finite rate and proposed a "zone of action" within which the significant effects are confined. Through use of momentum conservation and an "effective mass" of the target plate, they

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determined the resisting force, deceleration and penetration of a nondeforming conical projectile striking a thin plate at normal incidence. The analysis was later extended to cover truncated conical and ogival projectiles [50] and truncated cones at oblique impact angles [51]. A pitfall of their otherwise elegant approach lies in the requirement that the deformation pattern for the target be assumed a priori, requiring a good deal of insight on the part of the user of their models. Computed velocity-distance histories compare fairly well with experimental data quoted (10-25% discrepancy).

PYTEL and DAVIDS [52] consider deformation of a viscous plate by a rigid projectile. The plate is assumed to be acted upon by an initial velocity over a circular area with a radially symmetric shearing stress, uniform across the plate thickness, being the only non-negligible stress component. The theory requires a viscosity coefficient and strain rate for prediction of displacement fields. MINNICH and DAVIDS [53] modified the theory by including an empirically determined "impact yield constant" which has units of stress. Below this value the target is assumed to act as quasi-rigid whereas above it viscous flow occurs. This modification to the model leads to finite final deformed shapes but as the impact yield constant was found to vary with plug displacement the model is of no practical utility.

AWERBUCH [54] and AWERBUCH and BODNER [55-56] have analyzed the normal perforation of projectiles into metallic plates. The penetration process is assumed to occur in three interconnected stages with plug formation and ejection being the principal mechanism of plate perforation. In the first stage, shearing is assumed not to occur. This stage is considered to be a compressive stage in which the forces acting on the projectile are an inertial force and a compressive force. The inertial force is due to the acceleration of the mass of the target material in contact with the projectile in the direction of motion. The compressive force on the projectile is due to the compressive strength of the target material in contact with the projectile. Another basic assumption for this stage is that mass from the target material is added to the projectile during the penetration process.

The second stage of penetration is the onset of shearing of a plug from the target plate. In this stage of incipient plugging, the projectile is acted upon by the compressive and inertial forces of the first stage as well as a shearing force due to the motion relative to the target plate of target material which is accelerated by the projectile during this stage.

A third stage begins when plug and projectile move together as a rigid body with only a shearing force acting on the plug's circumference along its whole length. The theory allows computation of post-perforation velocity, force-time history and contact time for perforation processes that include dishing, plug formation and ductile cavity enlargement. However, several parameters in the analysis must be determined empirically,

namely the target entrance and exit diameters, the plug length, the coefficient of target viscosity and the width of the shear zone. For the latter, the authors cite the paper by CHOU [57] for an analytical expression. With the above quantities determined, experimental results for lead bullets striking steel and aluminum targets showed good agreement with predictions for post-perforation velocities and duration times.

AWERBUCH and BODNER [58] modified their normal perforation theory to include the effect of angle of impact for cases where perforation occurs without ricochet or projectile fracture. The primary modification consists of replacing plate thickness with an effective line-of-sight thickness for the target and adjusting force and momentum expressions in the analysis accordingly. Comparison of theory to experimental results for 0.22 caliber lead bullets striking aluminum targets ranging from 2-6mm in thickness showed fair agreement.

SIMPSON [59] proposes a model for penetration and perforation for a striking velocity range of 1.2-5 kms$^{-1}$. The model envisions penetration/perforation of a finite thickness plate to consist of an initial stage of penetrator hydrodynamic erosion. The second stage consists of continued penetrator erosion and onset of plate deformation assuming a plugging mechanism. In the remaining stages, the penetrator is assumed completely consumed and a plug ejected from the target plate. The target deformation stages follow closely the works of MINNICH and DAVIDS [53] and AWERBUCH [54]. The author was unable to find perforation data in the specified velocity regime and therefore presents comparisons only for thick target penetration, comparing model predictions for penetration depth and hole diameter with experimental data for long rods and mass focus slugs. Except for slug data, agreement is generally good. The report contains an extensive annotated bibliography and a computer program for the derived set of equations.

The deformation and perforation of thin plates resulting from the impact of spherical and conical projectiles has been examined by GOLDSMITH et al [60]. Further work was reported by CALDER and GOLDSMITH [61] and GOLDSMITH and FINNEGAN [62]. The latter is of interest since an assessment is made of the relative magnitudes of dishing (plate bending) and plugging based on strain gauge data acquired on each side of the target plate. It was found that at higher velocities the perforation mechanism changes from bending to punching.

LETHABY and SKIDMORE [63] consider plugging near the ballistic limit. When the striking velocity is much greater than the limit velocity the target absorbs just enough energy to cause a punching type failure and the rigid penetrator theories tend to give better results than in cases where the impact occurs at the ballistic limit. At such velocities, the target absorbs more energy which results in greater target deflection (bending). Their model, using the assumptions of membrane theory, also includes a plugging criterion based on a

critical angle of deflection in the target. The model is limited to very thin plates and low impact velocities. Predicted critical projectile velocities agree to within 10% of those determined from experiments with mild steel cylindrical rods striking mild steel plates at velocities of 38-170 ms$^{-1}$.

WOODWARD and deMORTON [64] compute critical velocities for plugging and a residual plug thickness and velocity. The model is based on energy balance and assumes that shear and frictional forces resist penetration. Comparison with experimental data showed good agreement for critical and plug residual velocities and poor agreement for plug thickness. On the whole, the model is better suited for computations with hard targets than with soft ones.

KOWALSKI et al [65] employ one-dimensional wave theory and assume shear failure to compute graphically the minimum striking velocity necessary to eject a plug. Although a numerical example is given, no comparison with experiments is made.

Many researchers concern themselves with the minimum velocity necessary to perforate a plate and the residual velocity of the projectile after perforation. The simpler models rely on an assumed failure mode and energy or momentum balance (or both) as well as a few well-placed empirical constants. NISHIWAKI [66] proposed a residual velocity model assuming that the total resistance to the motion of a rigid conical penetrator is a function of dynamic and static pressures. The assumption that displaced target material remains in contact with the projectile nose yields an expression for dynamic pressure. The static pressure is assumed to be a material constant. GABBERT [67] modified the Nishiwaki theory by assuming that particles of target material are displaced in a direction along the projectile trajectory with a velocity equal to that of the projectile rather than allowing target particles to displace normal to the projectile surface with a velocity equal to the component of projectile velocity in that direction. Both the Nishiwaki theory and Gabbert's modification were compared with a large body of experimental data. Both were found wanting.

RECHT and IPSON [68] develop a model for the residual velocity of a rigid penetrator using energy and momentum conservation and assuming plug formation to be the failure mechanism. The model requires an a priori knowledge of the minimum velocity required to perforate. They also suggest an expression for this velocity in terms of projectile diameter, length, density and sonic velocity and target shear strength, density, thickness and sonic velocity. GIERE [69] offers a residual velocity expression that differs little from that of Recht and Ipson. IPSON and RECHT [70] propose a means of determining minimum perforation velocities with a ballistic pendulum technique. WOODALL et al [71] offer considerable data on

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plugging and perforation at velocities where plug shattering can be expected. There is limited confirmation of the model suggested by one of the authors (Heyda). WEIDMAN [72-73] presents several approximate methods for calculating ballistic limits. His analysis is restricted to short cylindrical projectiles perforating thin plates or sheets. He assumes perforation to take place when the magnitude of the strain rate is less than some critical value and the magnitude of the strain is greater than another critical value. Shearing is assumed to be the dominant failure mechanism and the critical values for strain and strain rate are determined graphically in terms of a mass ratio factor. For thin sheets, perforation is assumed due to plugging, transverse shear stress is assumed constant through the plate thickness and target material is treated as a viscoplastic Bingham solid. Projectile mass is assumed small in comparison to the mass of the resulting plug and failure occurs when the radius of the hole is equal to the radius of the projectile. Using a series expansion for strain and strain rate given by CHOU [57] the above failure criterion is incorporated and truncated at the second term to get an explicit formula for ballistic limit. The two-term series shows excellent agreement with a full series solution (less than 5% discrepancy), especially at low velocities. No comparison with experimental data is made.

HEYDA [74] proposes a model for limit and residual velocities assuming plugging and resistance to projectile motion to be governed by two components of pressure, namely a high intensity component computed from hydrodynamic theory since it is assumed that this pressure causes a thin fluid zone to be formed at the plug-projectile interface and a second component of pressure resisting plug motion by shear. The shear stress is assumed constant through the plate thickness. It is further assumed that shear stress is the only component giving rise to mechanical work.

LEONE [75] develops a plugging model based on energy balance using an empirical relationship for limit velocity developed by BURCH and AVERY [76]. He compares residual velocity predictions of his own and nine other models ranging from the totally empirical to analytical. Two empirical models, his own and that of RECHT and IPSON [68] are found to give realistic results.

LAMBERT and JONAS [77] have reviewed the penetration theories dealing with non-deforming projectiles. They find that the variety of models proposed, such as the resisting force approach of Poncelet-Morin, the energy-momentum analysis of Recht and Ipson and other approaches due to Nishiwaki, W. T. Thompson, Zaid and Paul, etc. almost invariably adhere to one basic form, namely

$$V_r = \begin{cases} 0, & 0 \leq V_s \leq V_\ell \\ \alpha(V_s^2 - V_\ell^2)^{1/2}, & V_s > V_\ell \end{cases}$$

Here, $V_s$ is defined as the striking velocity, $V_r$ the projectile residual velocity and $V_\ell$ the ballistic limit,

$$V_\ell = \max \{V_s : V_r = 0\} = \inf \{V_s : V_r > 0\}$$

The various models reviewed ultimately differ insofar as do the formulations for $\alpha$ and $V_\ell$. Their examination of available data tended also to confirm that experimental results can often be well represented within the framework of the above form for residual velocity, particularly in situations where there is not excessive projectile deformation. Their report discusses in depth the characteristics of the form and generalizes it to

$$V_r = \begin{cases} 0, \ 0 \le V_s \le V_\ell \\[2ex] \alpha(V_s{}^p - V_\ell{}^p)^{1/p}, \ V_s > V_\ell \end{cases}$$

where $\alpha$, $p$ and $V_\ell$ are viewed as parameters to be subjected to optimal adjustment in a given situation. LAMBERT [78] has since provided equations for predicting the parameters $\alpha$, $p$ and $V_\ell$ for perforation of steel and aluminum targets by long rods.

DUNN and HUANG [79] have discussed impact on spaced plates, essentially summarizing existing equations for residual velocities based on plugging failure and estimate velocities and material parameters leading to projectile shatter based on simplistic shock wave considerations. ZAID and TRAVIS [80] offer an intelligent discussion of the subject together with a review of the pertinent literature and data for impact of hardened steel cylindrical projectiles striking single and multi-plate mild steel targets at velocities up to 500 $ms^{-1}$.

WILMS and BROOKS [81] approximate the projectile as both a rotary and a shear beam column to obtain transient bending and shear stresses in the projectile on oblique impact. Closed form solutions are obtained via Laplace transforms and complex boundary conditions obviated by assuming that the nose is embedded in the target and loaded hydrodynamically, not very severe approximations if the target thickness to projectile diameter ratio is greater than one (i.e., thick targets). Variation of shear stress with obliquity is studied. The authors find that the octahedral shear stress at 60 degrees will be some 3.3 times greater than that at normal incidence.

Attempts to determine frictional or heating effects have confounded many researchers. Both analytical and experimental results are mixed. WINGROVE [82] experimentally obtained force-time curves for blunt, hemispherical and ogival penetrators striking aluminum targets at

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velocities up to 240 ms$^{-1}$. He concludes that the maximum force is
independent of geometry although the deformation pattern and nature
of failure differ markedly for each of the penetrator shapes. He
also concludes that frictional effects are probably negligible
except in the vicinity of the ballistic limit and cites the results
of KRAFFT [83] who measured frictional adhesion between projectile
and target during ballistic penetration with a torsion type Hopkinson
bar. Krafft concluded that sliding friction amounted to at most 3%
of the projectile striking energy. This is in marked contrast with
the findings of MACH et al [84-85] and WEIRAUCH and LEHR [86] who
attribute temperatures in excess of 1200°C to surface friction
effects for penetrators passing through thick aluminum targets.
A previous determination of forces resisting penetration was made
by MASKET [87].

GORDON [88] attempted to use the Heyda model coupled with
heat transfer to determine the temperature distribution at any
point on an impacted plate. He assumes that heat is transferred
only in the radial direction and that the velocity of the plug and
projectile are the same. Comparison with experimental data indicated
that the model is not a valid one.

BACKMAN and FINNEGAN [89] provide an analytical procedure for
representing the behavior of a sphere-plate system solely in terms
of the motion of the sphere, whose trajectory is modelled as a series
of segments of constant speed and curvature. The resisting force is
taken to be of the Poncelet form (quadratic in velocity) and failure
of the target plate is modelled by removal of the force over pre-
determined areas once critical penetration depths are exceeded.
Comparison with experimental data tends generally to support the
model.

Qualitative discussions of adiabatic shear are given by, among
others, RECHT [90], STOCK and THOMPSON [91], WINGROVE [92]
and BACKMAN and FINNEGAN [93]. This subject is presently an item
of intense research with analytical models anticipated shortly.
Current work on containment of projectiles is that of ZAID, EL-KALAY
and TRAVIS [94]. The whole field of containment of ballistic
fragments has been reviewed by RECHT [95].

Essentially all thin plate perforation models deal with rigid,
non-deforming penetrators. Two exceptions are the models proposed
by HASKELL [96] and RECHT [97]. Both assume plugging failure
of the target. Haskell characterizes rod behavior in terms of one-
dimensional plastic wave propagation, a simple maximum strain
failure criterion and a conservation of energy treatment of the
perforation process. Recht follows concepts advanced by G. I. Taylor
and treats mass loss as a multi-stage process characterized by
differences between the relative velocity of deformed and undeformed

portions of the projectile and the materials' plastic wave velocity.
The Recht model predicts mass loss due to shock front, the lengths of
deformed and undeformed portions of the rod, the shape of the deformed
portion and the extruded mass left behind during perforation.  Both
models predict residual mass and length reasonably well in comparison
against finite data sets.


### III.  NUMERICAL METHODS

If a complete solution of impact problems is desired, recourse
must be made to numerical techniques, i.e., numerical solution of
the full equations of continuum physics.

Numerical methods are capable of attacking the entire set of
field equations, have greater flexibility than various algebraic
equations and can accurately model transient phenomena.  They are
still approximate in nature, but generally errors associated with
uncertainties in material properties are far greater than errors
inherent in the numerical method.

There are several distinct disadvantages, though, which must be
considered.  Information about the solution is obtained only at a finite
number of points and cannot be considered complete until the data has
been interpolated to get the entire flow field.  The number of mesh
points can always be increased but this inevitably increases costs by
significant amounts and generates more information than can be profitably
analyzed.

The use of numerical methods tends to obscure the effects of
individual parameters involved in the problem.  An algebraic solution
immediately highlights each parameter involved.  To get similar information
from numerical methods, it is usually necessary to repeat the calculation
many times, varying the parameters of interest.  There is also the
danger that the detailed work required to obtain a numerical solution
tends to obscure the real problem, i.e., sacrifices may be made to achieve
reasonable costs and running times so that the problem finally solved
is rather distantly related to the problem originally posed.

Perhaps an optimum use of computer codes would involve the
generation of several numerical solutions followed by the development
of approximate models based on observations of dominant characteristics
of penetrator-target interactions.  This in turn can lead to crude
algebraic solutions so that much can be learned about parameter variation
without a large expenditure of computer time.  In effect, the numerical
solution becomes a "guessed" solution for the approximate analytical model.
Unfortunately, the pressure of meeting deadlines can often be so great
that this vital step is never taken and only a continuous stream of
numerical "guesses" is generated.

Computer codes for impact studies fall into two categories:
Lagrangian and Eulerian. Lagrangian codes follow the motion of fixed
elements of mass and the computational grid is fixed in the material.
These codes are conceptually straightforward and have the ability to
handle boundary conditions at free surfaces and contact surfaces between
different materials. However, inaccuracies in the numerical approximations
grow when cells become significantly distorted due to shear and when
cells fold over themselves resulting in negative masses. At this point,
recourse must be made to rezoning. While rezoning techniques have
been used quite successfully in one-dimensional codes, especially to
increase definition in regions where physical quantities vary rapidly,
even the most complicated and sophisticated rezone routines have been
disappointing for the two-dimensional case, with the apparent exception
of the TOODY code.

In the Eulerian approach, the grid is assumed fixed in space and
the continuum passes through it. Material can be represented as
either discrete points or continuously. Such codes can handle large
flows with large distortions but may be limited by not accurately
calculating free surface motion or conditions at material interfaces
unless Lagrangian features are incorporated.

The use of such codes is neither straightforward nor inexpensive.
A typical code will output about $10^7$ words of information and cost upward
of $1000 per run. Of necessity, much of the output is presented in
plot form. Considerable experience is required to run the codes and
frequently manual intervention is called for. In no way can present
codes be treated as "black boxes".

The dominant discretization schemes used in most computer codes
capable of treating penetration problems are the finite element and
finite difference methods. Convected coordinate and hybrid methods
are also discussed in an excellent review article by HERRMANN [98].
Some interesting ad hoc numerical methods which do not fit into the
above categories are mentioned below.

Numerical techniques applicable to studies of high speed impact
phenomena are discussed in depth by WALSH [99], HERRMANN and HICKS [100]
and HERRMANN, BERTHOLF and THOMPSON [101]. HERRMANN, HICKS and YOUNG [102]
consider one-dimensional methods applied to stress-wave attenuation
in solids. MESCALL [103] has reviewed the successes and limitations
of one and two-dimensional codes for penetration and shock propagation
problems. Von RIESEMANN et al [104] also review one- and two-dimensional
codes. Hence, the following remarks will be addressed primarily to
current developments in three-dimensional computations of impact response
of solids. Two interesting diversions will first be considered.

RAFTOPOULOS and DAVIDS [105] developed and propounded in a series of reports [106-111] a method relying on a sequential solution of the equations governing material impact (rather than manipulating them into differential equation form and then using standard numerical techniques) for a given time and finite region of the geometry considered. Computations were advanced in time and space until a "solution" was obtained. A variety of impact problems was considered and comparisons made with experimental data. Results for impacts into metallic targets were mixed and there was an indication that the procedure was sensitive to the order in which the equations to be solved are arranged. ROYLANCE, WILDE and TOCCI [112] ressurected the scheme and applied it to the analysis of textile body armor impact with some success. An approach similar in spirit, though less complex, was employed by VINSON and ZUKAS [113] to study impacts into nylon and Kevlar targets, also with reasonable experimental correlation.

DONALDSON et al [114-115] propose a one-particle model for penetration satisfying all global conservation equations and assuming that a drag force is the mechanism for penetration resistance. Their computer code is designed to predict trends only rather than provide single point correlations. That goal is generally achieved. Projectiles are idealized as non-deforming spheres and rectangular particles which can either deform or remain rigid. The model requires two empirical parameters, a characteristic energy and velocity. The former is the amount of energy required to cause hydrodynamic flow in the target while the latter is interpreted as a measure of elastic energy storable in a target. Experimental data for a variety of materials is presented for determination of the empirical parameters. A theory is also presented for relating characteristic parameters to material properties, such as Brinell hardness, determinable from static tests. Viewed optimistically, the theory is qualitatively correct.

The classic paper by WILKINS [116] served as a stimulus towards development of large scale computer codes for study of hypervelocity and ordnance velocity impact phenomena. Currently, a number of three-dimensional codes are available. With exceptions to be noted below, they are based on their two-dimensional predecessors, the general features of which are described in the review articles by Mescall, Herrmann and Von Riesemann et al mentioned previously.

The Lagrangian finite difference code HEMP3D, developed by WILKINS et al [117], is an outgrowth of the two-dimensional HEMP code and is designed to solve problems in solid mechanics involving dynamic plasticity and time-dependent material behavior. It is based on an incremental formulation for elasto-plastic behavior, employs the von Mises yield criterion and relies on artificial viscosity for diffusion of steep shock fronts. The code has been applied to a variety of static and dynamic problems, including fracture [118-119] It is being extended to include sliding surfaces for treatment of penetration problems.

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TRIOIL and TRIDORF are both Eulerian three-dimensional finite difference codes, developed by W. E. JOHNSON [120-121]. They have been applied to the study of shaped charge penetration of finite thickness plates (with the jet modeled as a rigid rod) at high obliquity as well as other problems. Both codes are similar to their predecessors, OIL and DORF, except that TRIDORF is a two-material code with a rigid plastic strength formulation. Similar in spirit is METRIC, developed by HAGEMAN et al [122-123]. The numerical methods and material descriptions are similar to those employed in HELP, its forerunner. The code is not core-contained so that in theory it can provide any degree of spatial resolution. This feature is necessary since, unlike HELP, the code lacks Lagrangian massless tracers to follow material interfaces and relies instead on mixed cells (cells containing more than one material) to establish material boundaries, thus making solutions highly grid-dependent.

Conventional finite element methods (those based on variational techniques and employing a stiffness matrix) based on Eulerian material descriptions are under development by REDDY [124] and CHAN et al [125] The latter have developed models that include viscoplastic and strain rate effects and account for material failure. Impact is viewed as a problem in the structural response class and the ultimate goal is the coupling of the Eulerian impact model with a Lagrangian structural response code such as NASTRAN.

G. R. JOHNSON [126-129] has taken a novel approach in developing two- and three-dimensional Lagrangian finite element codes (EPIC2 and EPIC3). EPIC3 (Elastic-Plastic Impact Calculations in 3 Dimensions), a three-dimensional computer code for dynamic analysis of high-velocity impact problems, is based on a Lagrangian finite element lumped mass formulation with the equations of motion integrated directly rather than through the traditional stiffness matrix approach. Nonlinear material strength and compressibility effects are included to account for elastic-plastic flow and wave propagation. The code has material descriptions which include strain hardening, strain rate effects, thermal softening and fracture. Geometry generators are included to generate quickly flat plates and rods with blunt, ogival or conical nose shapes. The technique is formulated for a tetrahedron element which is well suited to represent the severe distortions which occur during high-velocity impact.

Despite the existence of three-dimensional codes, numerical solutions of oblique impact problems are still few in number. Even on such fast computers as the CDC 7600, running times for three-dimensional codes are measured in terms of hours, even for coarse grids. Rather than review here the existing calculations which have been performed with the aforementioned codes, we refer the reader to the cited literature and in the next section present two solutions to oblique impact problems obtained at the Ballistic Research Laboratory (BRL) which are representative of the problems tractable by existing codes.

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## IV. OBLIQUE IMPACT SIMULATION

To illustrate current capabilities of finite difference and finite element codes in dealing with ballistic impact situations at high obliquity, two representative calculations are presented and compared to experimental data, the first involving penetration, the second ricochet. Geometric and material parameters for the two computations are listed in Tables II and III, respectively.

The penetration of a staballoy (depleted uranium) rod into a rolled homogeneous armor (RHA) plate at 60° was computed with both the plane strain version of the HELP code (a 2D Eulerian finite difference calculation) and the EPIC3 code (a 3D Lagrangian finite element calculation). Three dimensional codes have been successfully used on various problems, as noted previously, but they make severe demands on computer storage and are quite costly, though this latter aspect may become a minor problem with the advent of parallel processors. Hence, in the past, plane strain approximations have been used to obtain at least a qualitative appreciation of the behavior of rod and target under oblique impact conditions, which are clearly of great practical importance and sufficiently different from the normal impact case due to the added complexities of severe bending and asymmetric loading to warrant separate consideration. Two-dimensional plane strain calculations are straightforward enough, relatively inexpensive and provide some interesting information. At sufficiently early times, they can even be quantitatively correct. It must be recognized however that when oblique impact of an ogival projectile is treated as the impact of an infinitely long wedge important physical phenomena are being neglected not the least of which are the out-of-plane motions leading to lateral stress relaxations. Useful qualitative information about the early stages of an oblique impact can be obtained from plane strain solutions. Their utility degrades with increasing time after impact, however, so that for late times, when important aspects of penetrator and target response are being determined, plane strain solutions can be speculative at best.

As an example, consider the deformation field around an ogival (frictionless) penetrator. In the plane strain approximation, this would be modelled as an infinitely long wedge. A deformation front (the boundary between elastic regions and regions of permanent plastic deformation) which may or may not be attached to the apex of the wedge, moves with the wedge as it penetrates. As this front moves across an element of rectangular cross-section running parallel to the wedge, it will distort the cross-section in shear and start it translating laterally, possibly with a small vertical motion. Sectional distortion will continue until the sides of the element have been made parallel to the sides of the wedge, at which point penetration would proceed without any further distortion of that element. It should be subjected only to translation as there is now no geometric requirement for further distortion, i.e., steady state deformation will always occur in the mode requiring the smallest energy input.

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## Table II.  Penetrator and Target Characteristics

| Problem | Rod Material | Nose Shape | D | L/D | W | Target Material | T | θ | $V_s$ |
|---------|--------------|------------|---|-----|---|-----------------|---|---|-------|
| | | | (cm) | | (g) | | (cm) | (deg) | (kms$^{-1}$) |
| Perforation | Staballoy | hemi-spherical | 0.767 | 10 | 65 | RHA | 1.91 | 60 | 1.5 |
| Ricochet | VIMVAR S7 steel | " | 1.024 | 10 | 65 | HH | 1.27 | 75 | 0.946 |

D — projectile diameter
L/D — projectile length-to-diameter ratio
W — projectile weight
T — target thickness
θ — target obliquity measured from the normal to the surface of the target
$V_s$ — striking velocity

staballoy — depleted uranium-3/4 titanium alloy
RHA — rolled homogeneous armor
HH — high hardness armor

## Table III.  Material Properties

| Material | E | ν | $\sigma_y$ | $\sigma_u$ | ρ |
|----------|---|---|-----------|-----------|---|
| | (GPA) | | (GPA) | (GPA) | ($10^3$ kg/m$^3$) |
| Staballoy | 195.8 | 0.203 | 1.036 | 1.45 | 18.62 |
| VIMVAR S7 steel | 206.8 | 0.3 | 1.434 | 2.68 | 7.8 |
| RHA | 206.8 | 0.3 | 1.220 | 1.35 | 7.8 |
| HH | 206.8 | 0.3 | 1.434 | 1.744 | 7.8 |

E — Elastic modulus
ν — Poisson's ratio
$\sigma_y$ — yield strength
$\sigma_u$ — ultimate strength
ρ — density

Now consider the deformation surrounding a frictionless ogive. Again the deformation front moves with the ogive. However, the element under study is now a toroid of rectangular cross-section which is concentric with the ogive. When the front passes the element, the cross-section is distorted and the element is translated radially. Because of this radial displacement, the cross-sectional area of the element must be reduced in order to satisfy continuity. Thus, so long as penetration continues, that element will be both displaced and distorted. This is a fundamental difference between the plane strain approximation and the exact (axisymmetric) computation of penetration and is sufficient reason to expect different energy-displacement relationships for the two modes. The requirement for continuing distortion of all material within the plastic-elastic boundary clearly suggests that axisymmetric penetration should be a higher energy deformation mode than plane strain (wedge-like) penetration.

Figures 5 and 6 show the comparison between plane strain computational results and radiographs obtained at the PHERMEX facility of Los Alamos at 12 and 25.6 microseconds. (PHERMEX is essentially a 6 MeV x-ray source capable of shining through six inches of steel. It is invaluable for penetration studies since for the first time it is possible to obtain information about penetrator deformation and orientation within the target rather than having only initial and post-mortem data obtained with standard 150-300 KeV x-ray facilities.) Figures 7 and 8 show EPIC3 results for the same situation and Figure 9 depicts results for the two-dimensional Eulerian finite difference plane strain computation and the three-dimensional Lagrangian finite element computation and experiment at the later time. On the whole, agreement between both computations and experiment is remarkably good. The plane strain computation was performed on a UNIVAC 1108 computer with some 4300 mesh points and required five hours of computing time. The three-dimensional calculation used 4900 nodal points and some 23000 elements. It was performed on a CDC 7600 and required some seven hours.

Computations were also made to illustrate the capability of the EPIC3 code to handle the large distortions and severe bending often encountered in ballistic impact situations. Figure 10 is a schematic of the test setup at the small caliber range of the Terminal Ballistics Division of the BRL. Figure 11 is a radiograph obtained at that facility which shows the deformation of an AISI-S7 VIMVAR (Vacuum Induced Melt Vacuum Arc Remelt) processed steel penetrator striking a high hardness armor plate at 75° obliquity at approximately 60, 195, 395 and 545 microseconds after impact. Figure 12 shows computed profiles at the first two times. Agreement at 60 μs is perfect. There is some slight deviation between computed and experimental profiles at 195 μs however. We attribute this to the fact that in its present form EPIC3 uses a total elastic and incremental plastic formulation which precludes proper treatment of elastic unloading. A version of EPIC3 using an incremental elastic-plastic formulation is under
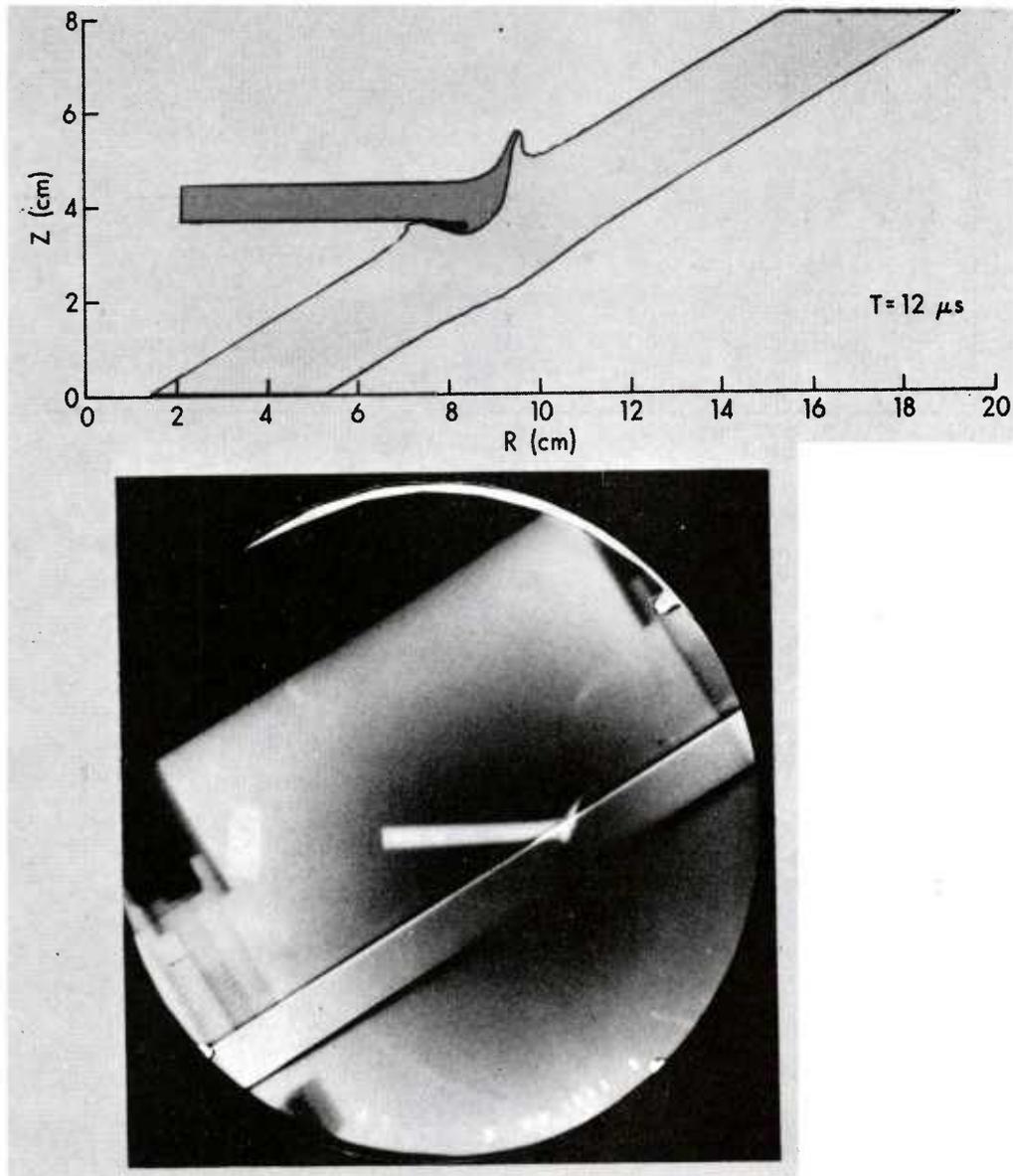
Figure 5.   Comparison of Plane Strain and Experimental Results at 12 μs
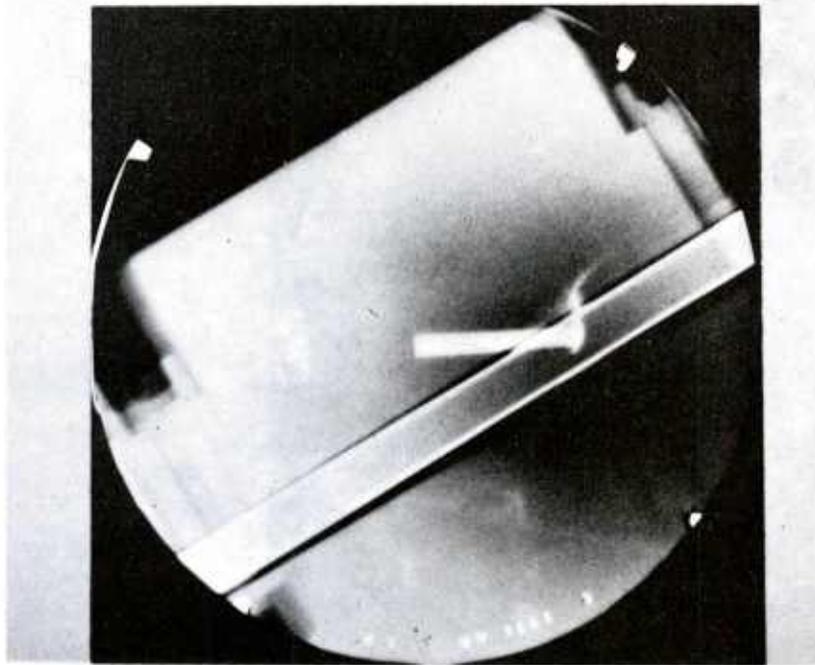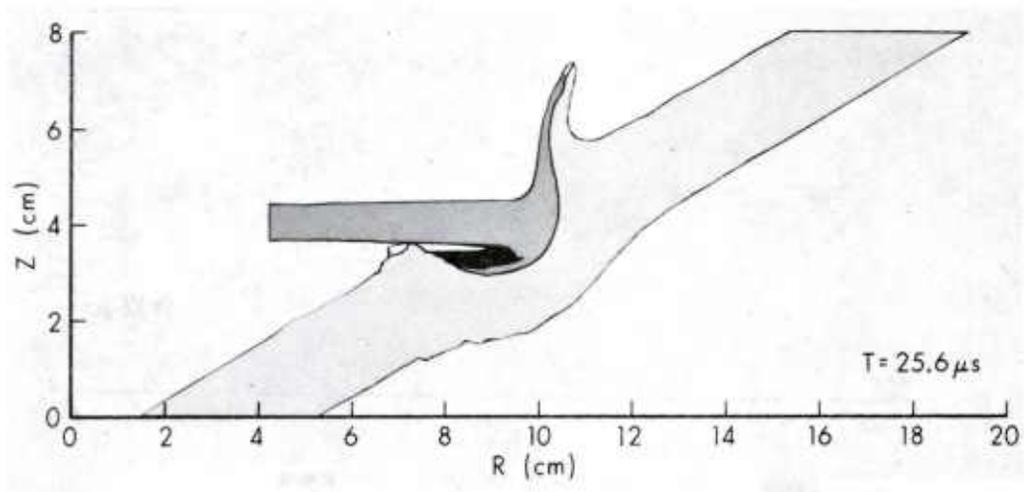
Figure 6.  Comparison of Plane Strain and Experimental Results at 25.6 μs
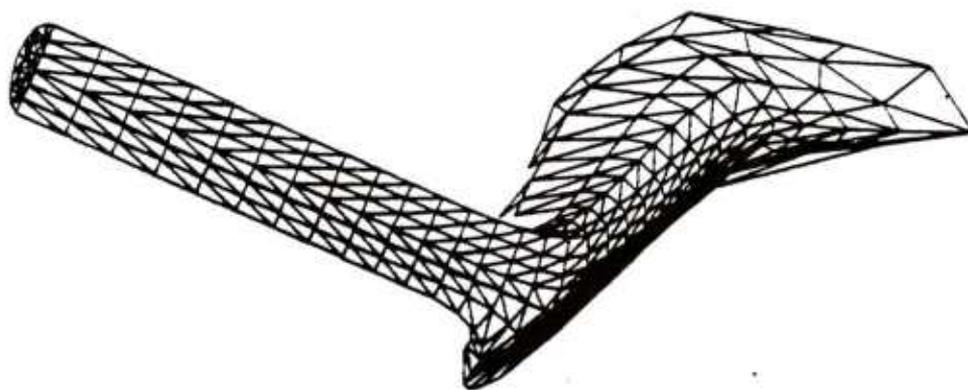
Figure 7.   EPIC3 Prediction of Penetrator Deformation at 25 μs
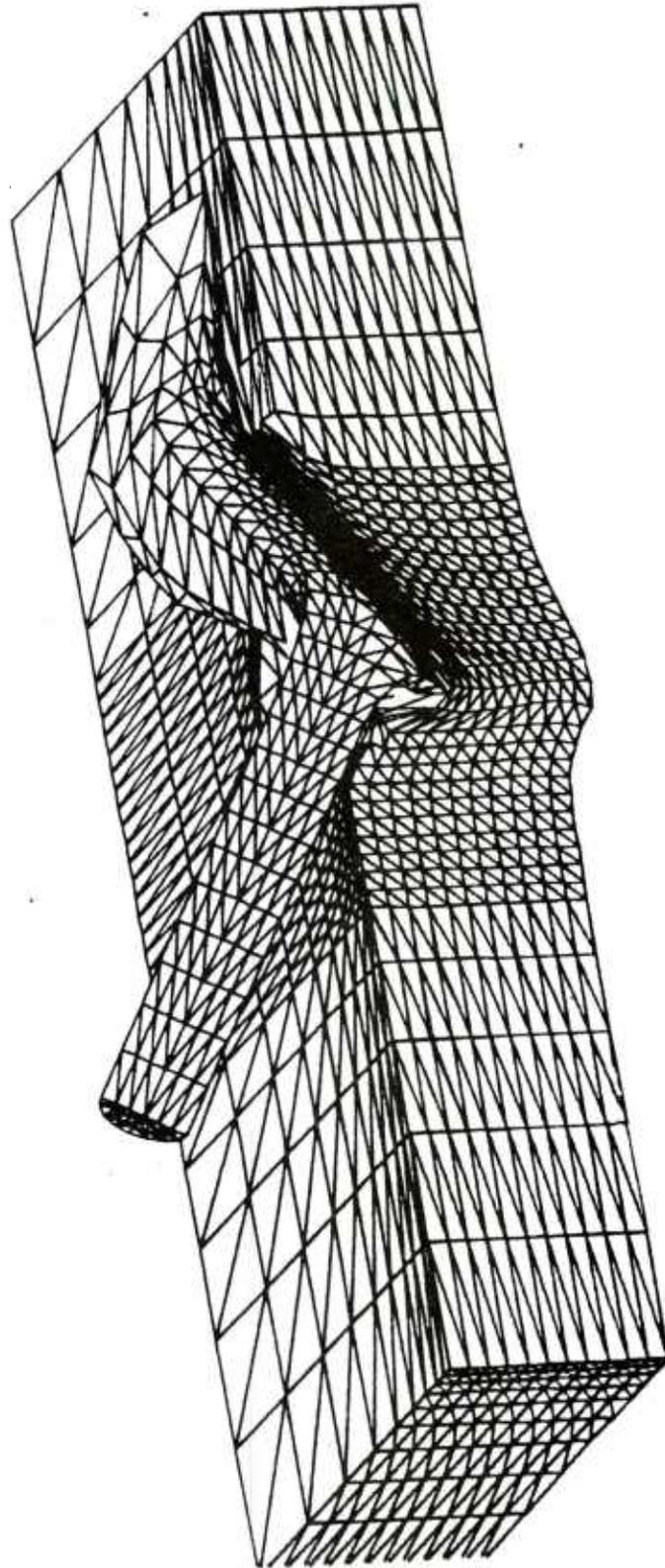
Figure 8. EPIC3 Predicted Target and Penetrator Deforamtion at 25 µs
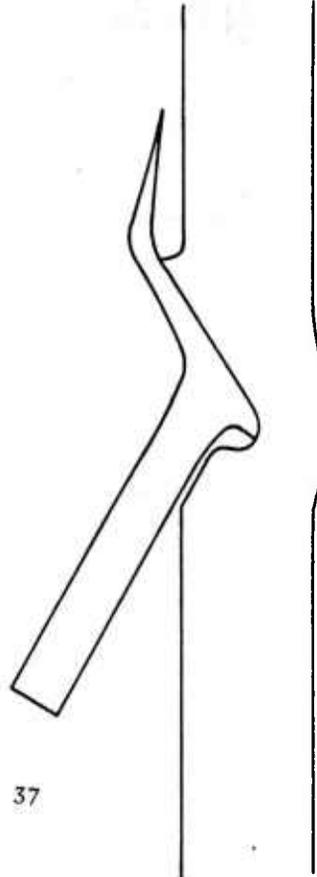
EXPERIMENT

HELP (PLANE STRAIN)

EPIC-3

Figure 9. Comparison of Experimental, Three-Dimensional and Plane Strain Results at 25 μs

OVERHEAD TUBES
FRONT OF TARGET | REAR OF TARGET

.020" DURAL WITNESS PLATE

CELOTEX RECOVERY

12"

72"

30"

72"

TARGET

REFERENCE WIRE

PROJECTILE
FLIGHT

FOUR 14" x 17" FILM

FRONT OF TARGET | BEHIND TARGET
SIDE X-RAY TUBES
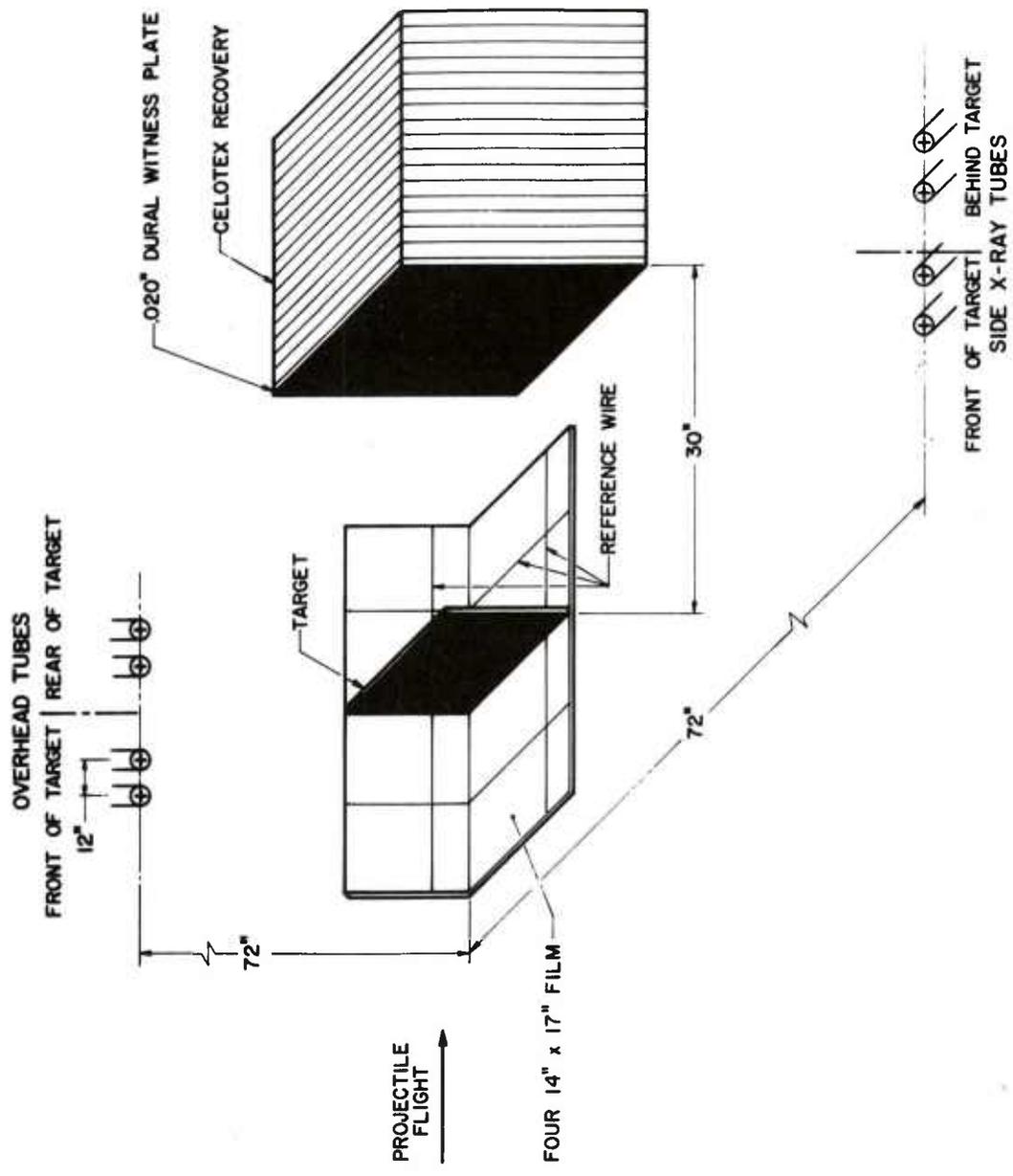
Figure 10. Schematic of Test Facility

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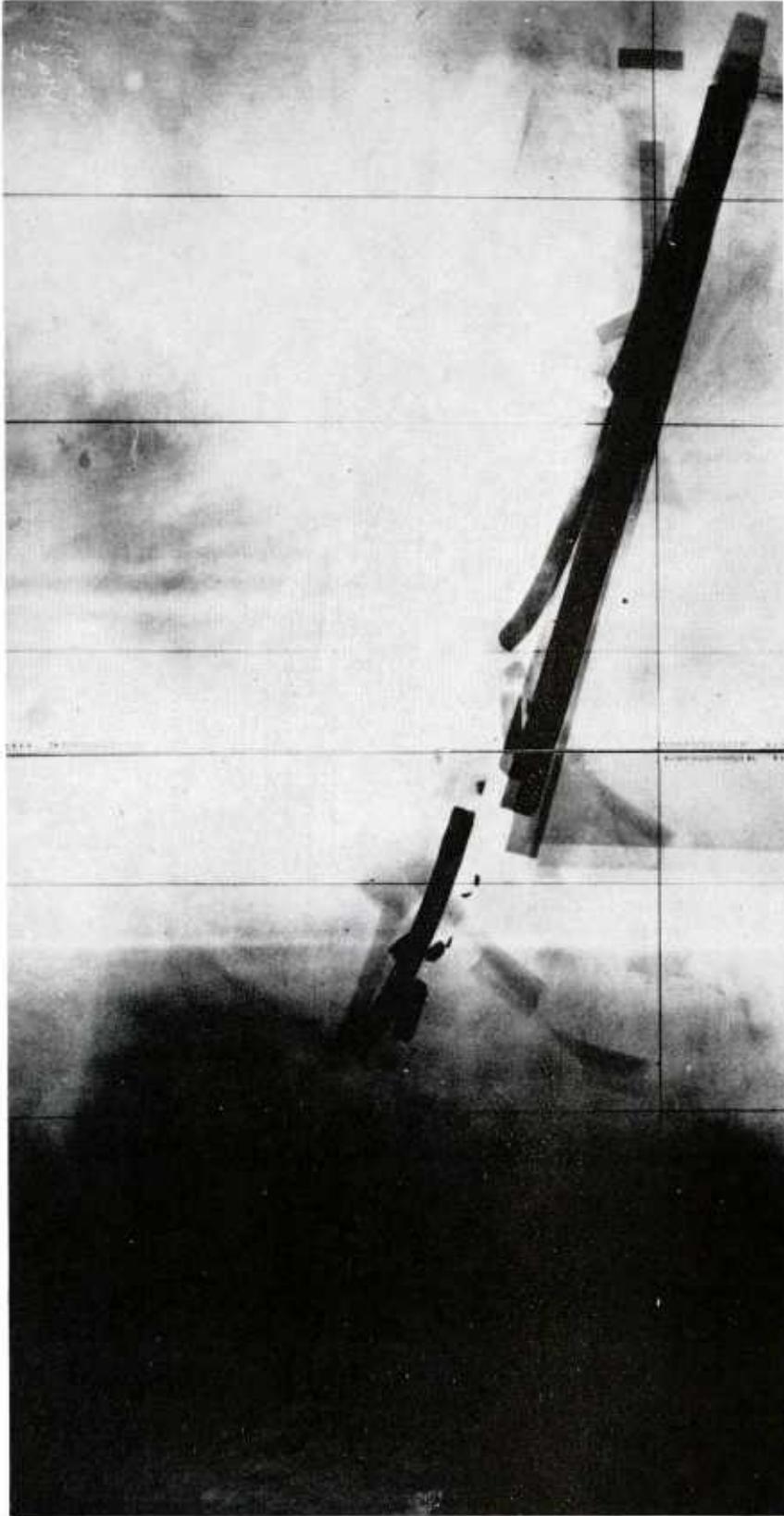
Figure 11. Radiograph of Penetrator Ricochet
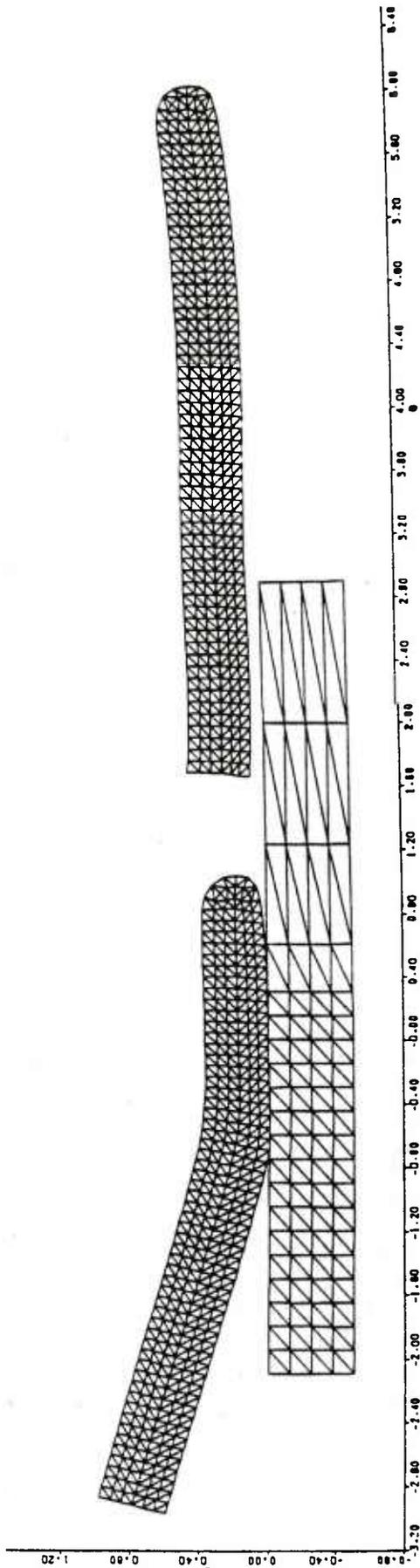
Figure 12. EPIC3 Predictions of Penetrator Deformation at 60 and 195 μs

development and will be available shortly. The computation was again performed on a CDC 7600 with 2343 nodes and 9360 elements and required some 2.4 hours of computer time.

Several points are worth emphasizing. None of the EPIC3 computations required rezoning. Such behavior for a Lagrangian code of any dimensionality in the face of deformations depicted in Figure 8 is nothing short of astounding. To the authors' knowledge, no other Lagrangian code is capable of such feats. The results fully justify the assertion by JOHNSON [129] that tetrahedral elements are ideally suited for impact computations. The running times, while still long, are feasible and are far less than normally anticipated for three-dimensional codes. And finally, contact and separation of impacting bodies in EPIC3 occurs automatically based on the loads experienced at the contact surface between the bodies and requires no user intervention or ad hoc specification of void opening or closing conditions. This is again a departure - a most satisfying one - from other codes for impact studies where contact between two bodies is maintained unless separation (or creation of new contact surfaces) is specified by the user.

As promising as these results appear to be, much work remains to be done in refinement of impact computations. A situation which none of the existing codes can handle is depicted in Figure 13 which shows a 75° impact of the same rod and target as in the previous case but at a higher velocity (1.082 kms$^{-1}$). While existing codes might indicate high stress areas which are likely candidates for occurrence of fracture (this would be highly dependent on the fracture model used), none can handle the fragmentation of an initially intact penetrator or target into individual fragments and then track the motion and deformation of those fragments. Such a capability is still far off and will require refinements in materials characterization, failure initiation and propagation. These topics are addressed in the following sections.

## V. MATERIAL FAILURE CHARACTERIZATION

It is now generally well established that failure of materials subjected to impulsive loads is a time-dependent phenomenon, e.g., papers presented at various conferences [130-132] as well representative papers by LUNDERGAN [133], HSIAO [134], KINSLOW [135], KALTHOFF and SHOCKEY [136]. With few exceptions, however, existing codes deal only with simplistic criteria which assume instantaneous failure of the material in a computational cell or element once a critical stress or strain is exceeded. Since more realistic failure models are at hand, it seems appropriate to briefly scan the work relevant to description of materials failure under ballistic impact conditions.
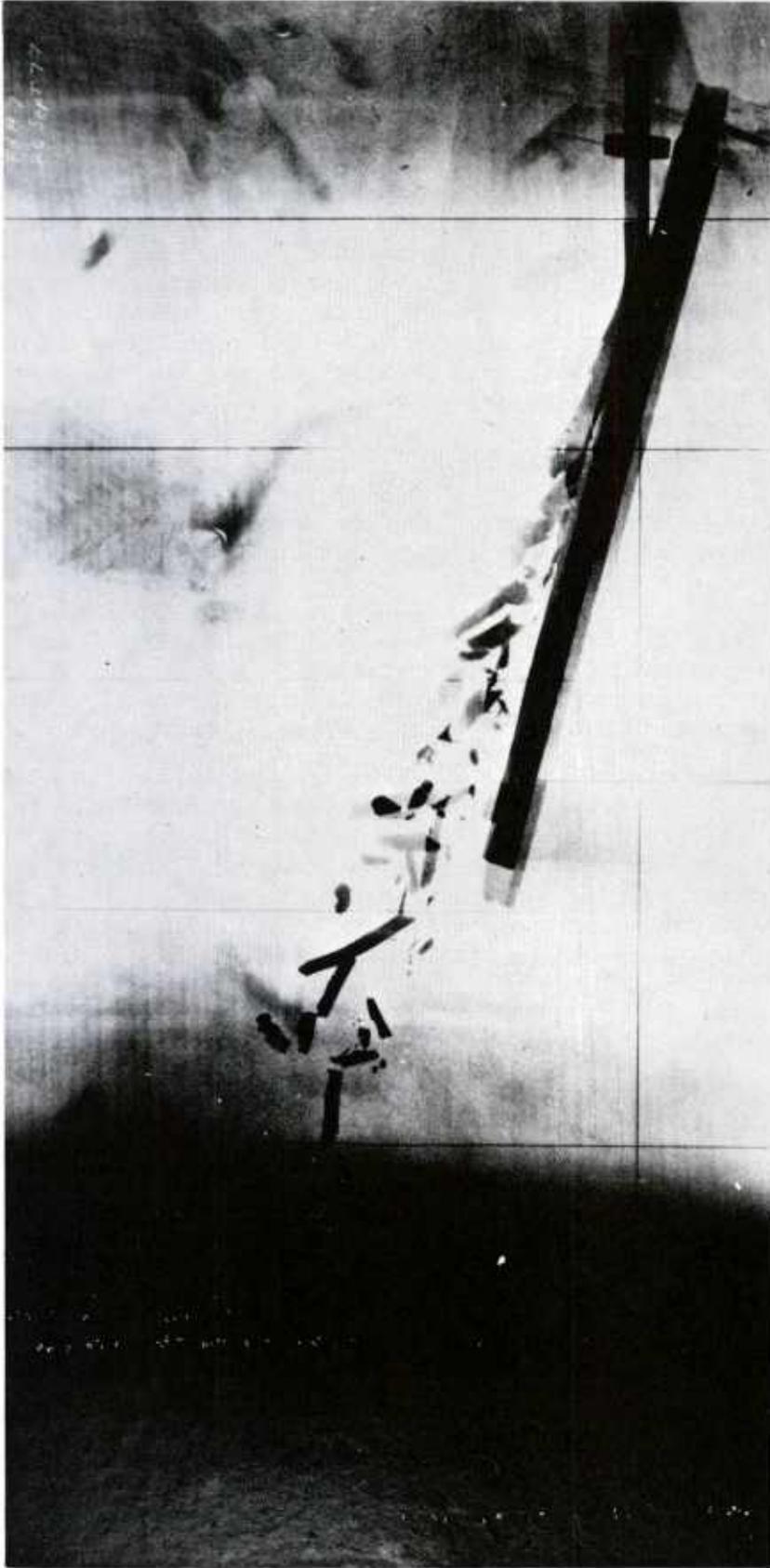
Figure 13. Radiograph of Penetrator Fracture

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By and large, available analytical models address the problem of spall formation. Spallation is material failure resulting from the tension produced by the interaction of rarefaction waves following impulsive loading. RIEDER [137] surveyed the literature on the theory and mechanisms of spallation. A thorough review and discussion of theories underlying large amplitude wave propagation and spall fracture under conditions of uniaxial strain was made by OSCARSON and GRAFF [138]. A recent article by KOLSKY [139] expands on the subject.

Except for very simple pulse shapes, analytical predictions of spall are best handled by wave propagation computer codes. The more sophisticated models, incorporating such parameters as time, rate, temperature etc. are by and large phenomenological in nature in that spall stress is related to time, strain rate, etc. by experimentally determined constants. An exception to this, the spall model developed by Waldorf based on atomic bond dissociation, is discussed in detail by Oscarson and Graff.

We can distinguish between three distinct approaches to fracture characterizations: continuum methods based on cumulative damage concepts, models concerned with microscopic aspects of ductile and brittle fracture and crack propagation models. The latter will not be considered here. The literature in this area is quite extensive. Papers by VITEK and CHELL [140], STEVERDING and LEHNIGH [141], NUISMER and ACHENBACH [142] and SIH and EMBLEY [143] are representative of work being done in this area.

TULER [144] has reviewed continuum aspects of the fracture of homogeneous materials subjected to impulsive loading, critically commented on available models and recommended fracture analysis models for conceptual, preliminary and final designs of re-entry vehicle structures. Previously, TULER and BUTCHER [145] proposed a time-dependent spallation criterion in which a damage function K is taken to be function of the entire stress history. The criterion is stated in the form

$$K = \int_0^t (\sigma - \sigma_0)^\lambda \, dt$$

where $\sigma(t)$ is a tensile stress pulse of arbitrary shape, $\sigma_0$ a threshold stress level below which no significant damage will occur regardless of stress duration and $\lambda$ is considered a material-dependent parameter chosen to fit experimental data. K can be interpreted as a measure of damage which approaches a critical value when a specified level of microscopic damage becomes visible at a specified magnification. The parameters of this model $\sigma_0$, $\lambda$ and $K_{cr}$ are considered as material

parameters. The Tuler-Butcher model has attracted considerable attention and has been successful in predicting spall locations for a variety of impact situations. LAMBOURN [146] found that a form similar to the Tuler-Butcher model successfully predicted incipient spall damage in beryllium. MESCALL and PAPIRNO [147] found good correlation between HEMP code predictions with the Tuler-Butcher model and ballistic tests for the impact velocity necessary to cause spall fracture in steel plates impacted by blunt cylinders.

Spall criteria suitable for use with numerical computations (method of characteristics) are presented by THURSTON and MUDD [148]. Their method allows estimating spall criteria when experimental data are unavailable and when temperatures are other than those at which the data was collected. Comparison with flyer plate experiments shows their method capable of predicting the possibility of spallation, its approximate location and degree of severity.

RAE [149-150] uses a linear elastic model as well as a hydro-dynamic elastic model to treat the spall fracture problem. Here the target is treated as a compressible inviscid fluid up to the instant when the shock wave reaches the rear surface. Thereafter, classical linear elasticity equations are employed with the incident stress distribution given in terms of the pressure distribution behind the incident shock.

DAVISON and STEVENS [151] present a generalization of existing spall criteria embedded in a theory of continuous spall damage. The theory assumes that damage accumulated during early loading history affects the way in which damage is accumulated at later times. The theory is based on local continuum quantities such as stress, strain, strain rate, stress gradient, etc. and lends itself to incorporation in wave propagation codes. It is shown that the Tuler-Butcher model is a special case of the cumulative damage theory. A damage function and a function for the rate of damage accumulation must be empirically determined. With these functions at hand, the model is used to represent a diversity of existing data for impacts on copper and beryllium. Building on their previously stated concepts, DAVISON and STEVENS [152] presented a theory for spallation in brittle materials while DAVISON, STEVENS and KIPP [153] consider spallation in visco-plastic materials.

BARBEE et al [154], SEAMAN and SHOCKEY [155] and SHOCKEY et al [156] view fracture as a nucleation and growth process whereby microscopic incipient cracks and flaws are activated and grow according to experimentally measurable nucleation and growth rate functions. These functions are viewed as fundamental material properties and have been determined for a variety of materials. The model has been incorporated in the two-dimensional HEMP, HELP and SMITE codes and tested against data from flyer plate experiments. Computations involving armor-penetrator interactions for realistic ordnance materials and velocities remain to be done.

## VI.  SUMMARY AND CONCLUSIONS

Computer codes are invaluable for obtaining a qualitative picture
of penetrator and target deformation.  They provide details not
normally obtainable from ballistic experiments.  Yet, it is not unfair
to state that, on the whole, they have not improved our understanding
of penetration phenomenology, except possibly in the hypervelocity
regime.  The reasons for this are not difficult to understand.  Like
a great many ballistics experiments, code computations are frequently
undertaken with a particular project in mind.  Lacking extensive
graphics capabilities for interactive input and plotted output, most
codes generate hundreds of pages of numbers and minimal plots to be
digested by their human servants.  Being notoriously bad at such
synthesizing and often laboring under cost and time constraints, code
users extract the information needed to further the goals of a
particular project or answer specific design questions and consign
the output to permanent storage.  Hence, immediate needs are
served but little contribution is made to an overall understanding
of penetration mechanisms.

Notable exceptions to this sorry state of affairs, are such
institutions as Lawrence Livermore Laboratories, Sandia and Los Alamos
Scientific Laboratories, for whom time and money appear to be at best
third-order perturbations.  The lack of concern with mundane problems
is reflected in the generally high quality and meticulous care of
published computational results, e.g., [157-160].  Elsewhere, the
present state-of-the-art in computations is best summarized by
a paraphrase of Richard's Law, to wit: "One good guess is worth a
thousand computer runs".

Aside from the above considerations, codes in their present form
cannot be relied upon for quantitative data (except by accident) in
the ordnance velocity regime because realistic material properties and
failure models are presently lacking.  For the hypervelocity impact
regime, codes are very effective and produce reasonable quantitative
results, e.g., MISEY [161].  Here, the pressures generated in the
colliding materials far exceed their strength.  The principal factor
in characterizing the material becomes the equation of state.
Excellent work in this area has been done over the past few decades
and accurate equation of state data now exists for a wide variety
of materials and loading conditions.  Hence, good material characterization
produces good qualitative and quantitative results.

At ordnance velocities, however, the situation changes.  Here,
the pressures generated in armor-penetrator interactions are of the same
order as stresses in the material and decay rapidly in passing through
the material.  Hence, material strength dominates response computations.

Codes perform poorly in this velocity regime for several reasons:

(a)  Many codes lack sophisticated constitutive equations to characterize material behavior.  Most codes were developed in the heyday of hypervelocity impact and too often a simplistic strength model was tacked on to the codes as an afterthought.  The situation is changing with the advent of finite element codes for two- and three-dimensional response calculations, but, by and large, advanced  theoretical developments in constitutive modeling and wave propagation are not to be found in available production codes.

(b)  The behavior of penetrator and target is dependent upon the dynamic stress fields set up by the impact.  However, dynamic materials data is usually not available, especially for current penetrator candidate materials, i.e., staballoy, various tungsten alloys and hard steels.

(c)  Most codes employ simplistic failure models based on instantaneous maxima of field quantities or ad hoc criteria for particular materials and failure modes (such as the maximum plastic work criterion for plugging in HELP).  Yet materials failure is a time-dependent process, as mentioned previously.

It is necessary to add here that despite these limitations, code results for deformation fields often bear reasonable resemblance to those found experimentally.  This can be partially explained by noting that the codes at least begin with the correct ingredients, namely the conservation equations of physics, and also by the fact that computation of displacements implies integration of the underlying differential equations.  Integration, being a smoothing process, can quite frequently obscure deficiencies in material description, especially in highly energetic interactions.  The situation is just the reverse for computation of local quantities such as stresses and strains and few comparisons between computed and experimental values are to be found in the literature.

Fortunately, the means are at hand to improve in a quantum fashion the capability of codes to simulate real world phenomena and significantly better results can be expected within the next few years.  Constitutive equations for solids subjected to high intensity loading have attracted considerable attention and there is a large body of literature in the area, e.g., HERRMANN and NUNZIATO [162].  Not all applicable forms are equally convenient for application and this will be a significant consideration.  LEE [163] has considered the problem of plastic wave propagation extensively and concludes that for a realistic model finite deformation nonlinear elastic theory must be combined with considerations of finite plastic strain.  Moreover, thermomechanical coupling influences are important so that thermodynamic considerations must be included in the commonly purely mechanical plasticity theory.

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Modularity in codes will become increasingly important so users have the flexibility to readily change constitutive models. Many of the older codes do not readily lend themselves to modification and in some a simple change in constitutive relations or equation of state can become a major undertaking.

Refinements in theory will be to little avail however until characterization of materials at the strain rates appropriate to ballistic impact conditions (up to $10^5$ s$^{-1}$) is achieved, for errors in input will outweight any gains in modeling. Again, considerable work is being done in this area which is or soon will be directly applicable. LINDHOLM, YEAKLEY and NAGY [164] describe a high strain rate tension-torsion tester. Current models are capable of performing tests at strain rates approaching $10^3$ s$^{-1}$ and refinements to increase that rate are possible. HARGREAVES [165] discusses high strain rate testing of materials with emphasis on torsional impact test techniques. Thin-walled tubular specimens were used to obtain stress-strain curves for three high-strength steels in the range of strain rates from $10^{-2}$ to $10^2$ s$^{-1}$ which permitted test conditions from isothermal to adiabatic and the effect of thermal feedback on the deformation process to be characterized. Split-Hopkinson bar devices are routinely being used to characterize dynamic compressive behavior of materials at strain rates up to $10^2$ s$^{-1}$, e.g., NICHOLAS [166] and the many papers presented at the Conference on Mechanical Properties at High Rates of Strain. Considerable data, both static and dynamic, on a variety of materials for structural and ballistic applications is being generated at the Johns Hopkins University by Bell and his coworkers, e.g., BELL [167]. Thus partisans of any of the above methods will soon have available a body of data which, though still not at the high strain rates encountered in typical ballistic impact situations, will be substantially better than static data and will enhance code quantitative prediction capabilities.

The last and most difficult hurdle to overcome will be determination of reliable and computationally suitable models for the onset of fracture and the characterization of failed material. This is a difficult area and will require considerable research over the next decade at least. Impressive progress is being made at such institutions as Sandia and Stanford Research Institute. This is an area where a combined analytical/experimental approach is most likely to lead to models suitable for engineering applications.

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